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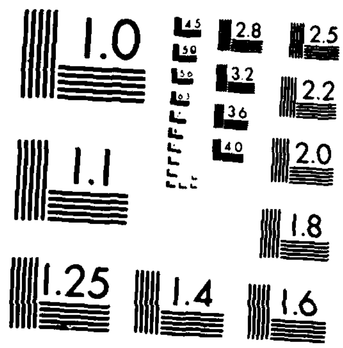

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OPTICAL COMPUTING STRATEGIES

Final

REPORT

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to

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The subject of the current research effort is the development of a theory of optical computing, it is generally agreed that optical computing has an advantage over digital computing in situations where parrallelism can be exploited. The canonical examples are matrix-vector multiplication and matrix-matrix multiplication. If the matrices are both square and of size (n x n) then outer-product decomposition achieves a saving in computational time because the N2 inner products can be evaluated concurrently. It is outlined in Section 1. Our second completed contribution is the development of a tractable mathematical model of an optical system (assuming incoherent light operations and its use into an investigation of the inherent limits of computation of such a system in terms of a lower bound on the simultaneous resources of volume and computing time.

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INTRODUCTION

Optical computing (that is the use of optical and electro-optical devices to perform mathematical computations such as matrix multiplication, solution of simultaneous linear equations, etc.) is a subject of current interest. One of the main reasons being the possible use of such device technology to large array processing, hopefully in the parallel processing mode. The computing speeds attainable using optical components are the major factor in this quest.

There are basically two aspects of the problem. The first is the physics and technology of the devices that perform the manipulations via optical and/or electro-optical means. A considerable effort has been expanded upon device development; it is safe to say that this program has now begun to bear fuition in that several devices have shown capabilities that warrant optimism.

~~This brings us to the~~ second aspect of the problem, the subject of the current research effort~~, namely~~ the development of a theory of optical computing, ~~For example,~~ it is generally agreed that optical computing has an advantage over digital computing in situations where parallelism can be exploited. The canonical examples are matrix-vector multiplication and matrix-matrix multiplication. Generally, investigators in optical computing, have taken algorithms directly from the standard numerical analysis literature and modified it for use in optical computing. The most successful example is matrix-matrix multiplication based on outer-product decomposition as popularized by Athale and associates. If the matrices are both square and of size $n \times n$, then outer-product decomposition achieves a saving in computational

time because the n^2 inner products can be evaluated concurrently. However, previous to the present investigation no one seems to have developed *ab initio* numerical algorithms specifically for use in optical computing by taking advantage of the fact that convolutions can be performed very rapidly. We have developed such an algorithm for matrix-matrix multiplications. It is outlined in Section 1. Our second completed contribution is the development of a tractable mathematical model of an optical system (assuming incoherent light operations) and its use into an investigation of the inherent limits of computation of such a system in terms of a lower bound on the simultaneous resources of volume and computing time. This material is outlined in Section 2. Note that the material in these two sections will be submitted for publication in the near future. Work is still continuing on the influence of device uncertainty on parallel processing via optical computing.

SECTION ONE

AN ALGORITHM FOR MATRIX-MATRIX MULTIPLICATION VIA CONVOLUTION

One of the virtues of electro-optical computing is the ability to carry out convolution operations very rapidly. Given this technical advantage, it is worthwhile to develop an algorithm for the multiplication of two rectangular matrices using convolution.

To this end let us consider the matrix product $\hat{C} = \hat{A}\hat{B}$ where $\hat{A}$ is of size $n_1 \times n_2$, $\hat{B}$ is of size $n_2 \times n_3$, and $\hat{C}$ is of size $n_1 \times n_3$. Let the corresponding matrix elements be a_{ij}, b_{jk}, and c_{ik}. Associate with $\hat{A}$ and $\hat{B}$ the polynomials $P(x)$ and $Q(x)$, with x being interpreted as an indeterminate

$$P(x) = \sum_{s=0}^{(n_1-1)n_2n_3+n_2-1} p_s x^s \tag{1}$$

$$Q(x) = \sum_{t=0}^{n_2n_3-1} q_t x^t \quad . \tag{2}$$

Note that the degree of $P(x)$ involves not only the size of $\hat{A}$ through n_1 and n_2 but also the size of $\hat{B}$ through n_3. The degree of $Q(x)$, on the other hand, involves only the size of $\hat{B}$, namely n_2 and n_3. The p and q coefficients are related to the matrix elements of $\hat{A}$ and $\hat{B}$ by

$$p_s = a_{ij} \,, \qquad \text{if} \quad s = (i-1)n_2n_3 + j - 1 \tag{3a}$$

$$= 0 \quad , \qquad \text{if} \quad (i-1)n_2n_3 + n_2 \leqslant s \leqslant in_2n_3 \tag{3b}$$

and

$$q_t = b_{jk}, \qquad \text{if} \quad t = kn_2 - j \tag{4a}$$

$$= 0 \quad , \qquad \text{if} \quad t \geqslant n_2 n_3 \tag{4b}$$

with: $1 \leqslant i \leqslant n_1$, $1 \leqslant j \leqslant n_2$ and $1 \leqslant k \leqslant n_3$.

We claim that the elements of the matrix product $\hat{C}$ are given by selected coefficients of the polynomial

$$R(x) = P(x)Q(x)$$

$$= \sum_{m=0}^{n_1 n_2 n_3 - 1} r_m x^m \tag{5}$$

where

$$r_m = \sum_{s=0}^{m} p_s q_{m-s} \tag{6}$$

is the discrete convolution of the p and q coefficients. These selected r_m are given by

$$r_m = c_{ik}, \qquad \text{if} \quad m = (i-1)n_2 n_3 + kn_2 - 1 . \tag{7}$$

A formal proof (which is really a verification of the formulae) is now given. We begin by rewriting Eq. (6) in the form

$$r_m = \sum_s p_s q_{m-s} = \sum_{\alpha,\beta,\gamma,\delta} a_{ij} b_{jk} \tag{8}$$

where the summation in the second series is over:

$$\alpha: \qquad s = (i-1)n_2n_3 + j - 1 \tag{9a}$$

$$\beta: \qquad (i-1)n_2n_3 < s < (i-1)n_2n_3 + n_2 \tag{9b}$$

$$\gamma: \qquad t = m - s = kn_2 - j \tag{9c}$$

$$\delta: \qquad t < n_2n_3 \tag{9d}$$

The α term is simply Eq. (3a), while the β term is the negation of Eq. (3b). The γ term follows from Eq. (4a), while the δ term is the negation of Eq. (4b). Upon substitution of the α term into the β inequality, we immediately see that this can only be true

$$1 \leqslant j \leqslant n_2 \tag{10}$$

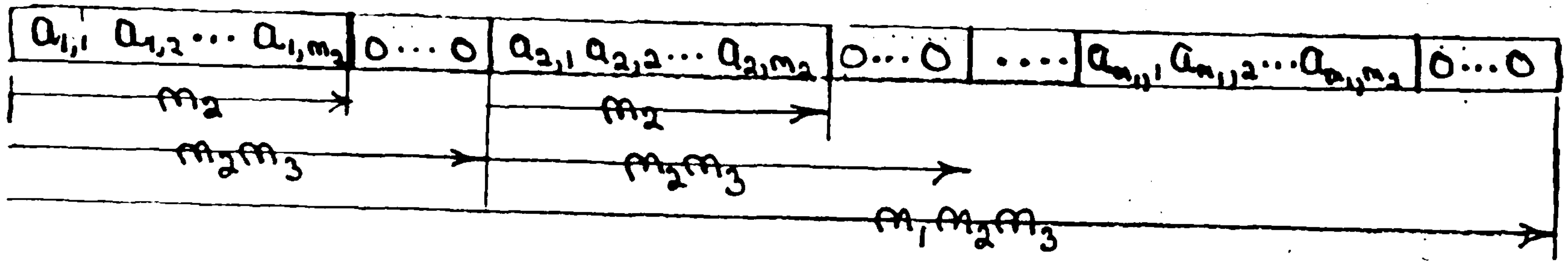
In like fashion, substitution of the γ term into the δ inequality leads to the requirement that

$$m = (i-1)n_2n_3 + kn_2 - 1 \tag{11}$$

which is Eq. (7). Thus the formulae are verified.

A construction which leads to the various formulae for p_s and q_t in terms of a_{ij} and b_{jk} respectively uses row vectors. Consider a row vector $\hat{p}$ whose elements we denote by p_s (coefficients of the polynomial $P(x)$) composed of the matrix elements a_{ij} of $\hat{A}$ and strings of zeros as depicted in Fig. 1A. The range of s is

$$0 \leqslant s \leqslant n_1n_2n_3 - n_2n_3 + n_2 - 1 \tag{12}$$



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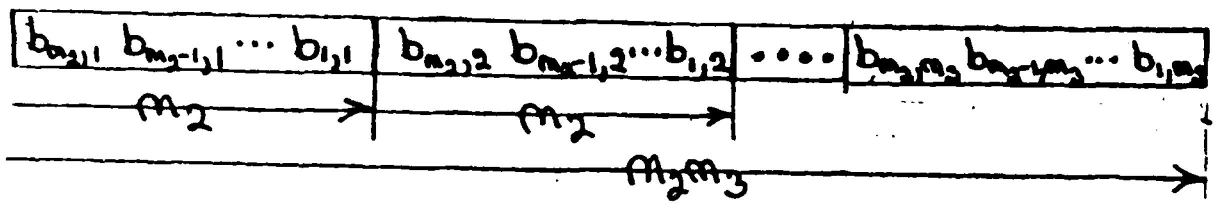

(B)

Fig. 1. Layout of the $\hat{p}$ vector, see (A), and the $\hat{q}$ vector, see (B).

consequently

$$p_s \equiv 0 \;, \qquad \text{if} \qquad s \geqslant (n_1-1)n_2n_3 + n_2 \tag{13a}$$

$$\equiv 0 \;, \qquad \text{if} \qquad s \leqslant in_2n_3 \quad . \tag{13b}$$

Furthermore the p_s are related to the a_{ij} as given by Eq. (3a), as the reader can verify by construction.

In like fashion, we construct another new row vector $\hat{q}$ with elements q_t according to Fig. 1B. Unlike $\hat{p}$, $\hat{q}$ has no strings of zero elements. The range of t is

$$0 \leqslant t \leqslant n_2n_3 - 1 \tag{14}$$

so that

$$q_t \equiv 0, \quad \text{if} \quad t \geqslant n_2n_3 \quad . \tag{15}$$

Within the range of t, the q_t are related to the b_{jk} by

$$q_t = b_{jk} \;, \qquad \text{if} \qquad t = (k-1)n_2 + n_2 - j \tag{16}$$

which reduces to Eq. (4a).

As an illustrative example of the algorithm, consider the case where $\hat{A}$ is 2×2, $\hat{B}$ is 2×3 so that $\hat{C}$ is 2×3 (i.e., $n_1 = 2$, $n_2 = 2$, $n_3 = 3$). The upper limits on the polynomials P, Q and R are 7, 5, and 11, respectively. The p_s, q_t and r_m coefficients evaluated according to Eqs. (3), (4) and (7) are listed in Table 1. Upon carrying out the convolution operation, Eq. (6), in conjunction with this table we have:

Table 1. Listing of the p, q and t coefficients for the case where $\hat{A}$ is 2×2, $\hat{B}$ is 2×3 and $\hat{C}$ is 2×3.

	p_s	q_t	r_m
0	a_{11}	b_{21}	
1	a_{12}	b_{11}	c_{11}
2	0	b_{22}	
3	0	b_{12}	c_{12}
4	0	b_{23}	
5	0	b_{13}	c_{13}
6	a_{21}		
7	a_{22}		c_{21}
8	0		
9	0		c_{22}
10	0		
11	0		c_{23}
12	0		

$$r_1 = c_{11} = p_0q_1 + p_1q_0 = a_{11}b_{11} + a_{12}b_{21} \tag{17a}$$

$$r_3 = c_{12} = p_0q_3 + p_1q_2 = a_{11}b_{12} + a_{12}b_{22} \tag{17b}$$

$$r_5 = c_{13} = p_0q_5 + p_1q_4 = a_{11}b_{13} + a_{12}b_{23} \tag{17c}$$

$$r_7 = c_{21} = p_6q_1 + p_7q_0 = a_{21}b_{11} + a_{22}b_{21} \tag{17d}$$

$$r_9 = c_{22} = p_6q_3 + p_7q_2 = a_{21}b_{12} + a_{22}b_{22} \tag{17e}$$

$$r_{11} = c_{23} = p_6q_5 + p_7q_4 = a_{21}b_{13} + a_{22}b_{23} \tag{17f}$$

These are, of course, the matrix elements as obtained by more standard procedures.

The implementation of the algorithm can be carried out in a straightforward fashion by re-examination of Figs. 1A and 1B. Note that the row vector $\hat{p}$ in Fig. 1A consists of the *rows* of $\hat{A}$ in which zeros are interspaced, the number of zeros is fixed. Thus we can easily handle the vector $\hat{p}$ containing the matrix elements a_{ij}. The row vector $\hat{q}$, containing the matrix elements b_{jk}, is simply the *columns* of $\hat{B}$ in reverse order, see Fig. 1B. This vector is also easily handled in the implementation.

SECTION TWO

LOWER BOUNDS ON THE COMPUTATIONAL EFFICIENCY OF OPTICAL COMPUTING SYSTEMS

The advent of *Very Large Scale Integrated* (VLSI) circuitry has lead to considerable decrease in the physical size of computers with a corresponding increase in speed of execution of operations. Basically there are three interrelated aspects to VLSI: design and fabrication of the chips, design of systems which use these chips for specific applications, and development of algorithms which utilize the inherent capabilities of such chips. The revolution in computer science, for both numerical and nonnumerical applications, brought about by VLSI continues unabated.

The computational limitations of VLSI were first investigated by Thompson [1]. For an introduction to this work see the basic text of Ullman [2] which contains references to subsequent work. It has been shown that any VLSI circuit with area A and time T requires at least $AT^2 = \Omega(n)$ to solve various computational problems such as FFT, convolution, and $\ell \times \ell$ matrix multiplication where $n = \ell^2$. The symbol Ω is defined in Ullman: $f(n) = \Omega(g(n))$ means that there exists a positive constant c such that for an infinite number of values of n we have $f(n) \geqslant cg(n)$.

Nevertheless, VLSI suffers from the limitation that the technology upon which it relies is inherently two-dimensional. Snyder's recent review [3] contains a very useful discussion of the constraints imposed by VLSI as regards planarity. In particular conventional VLSI chips are constructed by superposing a small number of layers on top of a substrate. This substrate has a thickness which is order of magnitude greater than the size of the transistors and wire width. Input and output from a conventional VLSI chip must be made

by a limited number of pads located on the sides of the chip. VLSI chip technology is changing almost daily; however, some of the more basic aspects are discussed in Barbe [4] and Einspruch [5]. Although an ensemble of two-dimensional chips can be placed on top of each other with holes drilled down through them for interchip communication, the total number of layers is seriously limited by the substrate thickness of each chip: consequently the resulting device cannot properly by termed "three-dimensional VLSI". For this reason, it appears that truly three-dimensional VLSI will most likely not be possible to fabricate. Nevertheless some interesting theoretical investigations of three-dimensional VLSI have been carried out: Rosenberg [6], Leighton and Rosenberg [7].

The purpose of the present communication is to summarize investigations into various aspects of the computational performance of three-dimensional devices which make hybrid use of electronic and optical components to perform operations. Our goal is to facilitate general statements on such electro-optical computations with specific reference to lower bounds on their complexity. Since such devices may contain a large number of components, we term them VLSIO, with the O denoting optics.

We note that a very useful overview of optical computing (more properly electro-optical computing) may be found in Caulfield *et al.* [8].

In order to carry out such an analysis we outline the development of an abstract model of VLSIO which is essentially technology independent but incorporates the physical restrictions of light beam propagation as expounded by Gabor [9], especially with respect to the very important fact that the

amount of information passing through a cube of small volume is bounded. This physical constraint allows us to adapt previous VLSI lower bound arguments to the VLSIO situation and allows for comparisons of electro-optical computing devices in terms of their volume V and the time T taken by VLSIO on a given input ($\equiv$ number of time units that elapse from the first input signal until the last output signal). We avoid making assumptions about the precise physics of the devices utilized. This would only limit the later application of these ideas as the physical models are improved and modified. Optical physics (through Gabor's theorem) implies an upper limit on the rate of information transfer across an optical beam, and hence a lower bound on computational efficiency of VLSIO. In addition we assume that any 2-D convolution of an $n \times n$ array of points can be achieved by a VLSIO device in unit time step. This assumption is reasonable because there already exist optical devices which perform thusly.

Note that all the variables and functions are taken to be Boolean (i.e., the values of the variables are taken from $\{0,1\}$).

We begin by discussing the well known abstract two-dimensional model of a VLSI chip as a $L_1 \times L_2 \times L_3$ grid graph with height L_3 ($<< L_1$ or L_2) held constant. The distance between grid points is w, the feature width. The chip processors are located at various distance nodes of the grid graph with each processor storing a state consisting of b bits. Furthermore the processors execute synchronously on a step consisting of a time unit of duration τ seconds. The remaining nodes are used for wire routing, or for input and output pods. Each wire can run along a path in the grid graph from

an input pod, or a processor, to various output pods, or processors. Wires are not allowed to intersect. On each time step, a value consisting of b bits of information is transmitted across the wire grid from either an input pod or a processor. The state of each processor is then updated on each step by a fixed function of the values transmitted by the wires leading into the processor, and by the state of the processor, in the previous step. The unit step transmission time across wires is justified by the fact that wire transmission can be made generally faster than transistor switching times. This remarkably simple model is sufficient to determine the computational efficiency of VLSI devices.

Following the two-dimensional version, the fundamental building block of our VLSIO device is the optical box B. It is a parallelopiped having lengths L_1, L_2 and L_3 with input and output faces, F_{in} and F_{out}. These faces are assumed to take as input and as output two-dimensional integer arrays $I(x,y)$ and $O(x,y)$ respectively. For convenience, we consider the input sources and output detectors to be very small compared to the size of the optical box (in order to minimize optical diffraction effects), furthermore they are uniformly spaced a distance w apart. The input sources are taken to be LED's (laser emitting diodes) and the detectors are unspecified except to state that they are sensitive only to the *intensity* of the LED radiation. We remaind the reader that most electro-optical computations are now performed via incoherent, geometrical optics based processors and not by coherent, Fourier transform based processors. The ancillary optical equipment (lenses, prisms, gratings, etc.) which spread and then collect the light can be

neglected in this version of the abstract model.

The output array is computed on each time step with a duration τ as a fixed function Δ_B of the input array; Δ_B will, of course, depend upon the detailed optical characteristics of B.

The optical box, in addition to being three-dimensional, also differs from VLSI in another way; namely, optical beams rather than wires provide storage and cross-flow. Since the *modus operandi* is incoherent radiation, these beams can intersect without interacting. The basic question that now arises is: "to what extent do optical (laser) beams behave as wires?"

A wire can only transport information at a finite rate depending upon wire cross-section, skin effects, etc. We would also expect an optical beam to perform similarly not withstanding the greater information rate. This problem has already been addressed by Gabor [9] who studied the "metrical information" in a light beam. The conclusion that he draws is that a light beam always has a *finite* upper limit with respect to information rates; the upper limit depending upon wavelength of light, smallest effective beam area, solid angle of divergence, etc. We need not concern ourselves with explicit formulae; for our purposes it suffices that we can interpret an optical beam as a wire.

Given this equivalence, we turn to the important problem of determining lower bounds (in terms of simultaneous volume and time) on the computational resources required for VLSIO to solve various problems.

In order not to unduly lengthen the text, it is assumed that the reader is familiar with Sections 1.4, 2.1 and 2.2 of Ullman's basic text [2].

Consider a Boolean function f with a set X of n input variables and a set Y of m output variables. Let X' be a subset of X; also let $P \equiv (X_L, X_R, Y_L, Y_R)$ where X_L, X_R and Y_L, Y_R are partitions of X and Y respectively. We term P balanced if between one-third and two-thirds of X' lies in X_L and note it by P_b. If α and β are two input assignments, then we term them a fooling pair of assignments to X if:

1) output Y_L is distinct for input assignments $\alpha(X)$ and $\alpha(X_L)\beta(X_R)$

2) output Y_R is distinct for input assignments $\beta(X)$ and $\alpha(X_L)\beta(X_R)$.

In addition, let the fooling set for P be a set of assignments A of X such that for all distinct $\alpha, \beta \in A$, at least one of (α,β), (β,α) is a fooling pair.

Finally, we require that the locations and times of the input and output are given only once.

Crucial to the analysis is the concept of information content (essentially "the amount of information that must cross a boundary in order to solve the problem"). Formally the information content of the Boolean function f is:

$$I_f = \max_{X'} \min_{P_b} \max_{A} \log_2(|A|) \tag{1}$$

where A denotes the fooling set corresponding to P_b. The following functions (of importance in electro-optical computing) are known to have information content $I_f = \Omega(n)$:

a) n point discrete Fourier transforms.

b) multiplication and inversion of two $\ell \times \ell$ matrices where $n = \ell^2$.

c) n point convolution.

The following important result on lower bounds is due to Thompson [1,2]: *Any two-dimensional VLSI chip computing a Boolean function* f *requires simultaneous area* A *and time* T *satisfying* $AT^2 = \Omega(I_f^2)$.

We now prove: *Any three-dimensional "optical box" computing a Boolean function* f *requires simultaneous volume* V *and time* T *satisfying* $VT^{3/2} = \Omega(I_f^{3/2})$.

The proof (which we now sketch) is an adaptation of the two-dimensional technique. Let the device be a parallelepiped having dimensions $L_1 \leq L_2 \leq L_3$ with volume $V = L_1 L_2 L_3$. Choose X' to be the subset of X such that $I_f = I_f(X')$. For each $i = 1,2,3$ we can find a cut C_i of area

$$A_i \leqslant 2V/L_i \qquad i = 1 \; ; \tag{2}$$

which disconnects the device into two components each of which contains at most two-thirds, but no less than one-third, of the inputs of X'. By definition at least I_f bits must be transported across each cut; this requires time

$$T \geqslant \frac{I_f}{A_i} \tag{3}$$

Consequently

$$V^2 T^3 \geqslant A_1 A_2 A_3 T^3 \geqslant I_f^{3/2} \tag{4}$$

or

$$VT^{3/2} = \Omega(I_f^{3/2}) \tag{5}$$

which is the sought-for result. The main point to emphasize is that this result depends upon the fact that we can treat light beams as if they were wires.

An immediate consequence of this theorem is that the lower bounds for optical computing are:

a) n point convolution or n point discrete Fourier transforms

$$VT^{3/2} = \Omega(n^{3/2}) \tag{6}$$

b) multiplication and inversion of two $\ell \times \ell$ matrices where $n = \ell^2$

$$VT^{3/2} = \Omega(\ell^3) \quad . \tag{7}$$

These results follow from the statements quoted after Eq. (1). Equations (6) and (7) represent the lower bound performance of these two operations in terms of volume and time. It is important to remember that these bounds are a consequence of the fact that we allow the entire volume of B to be operative.

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